A MODEL FOR THE DEMAND ON GATE 3

DANIEL G. DAVIS

ABSTRACT. After making the assumptions of our model explicit and allowing for 8 parameters, we give a formula for \( T \), the number of seconds that machine 3 needs to be on in order for the three machines to pack all the product that is produced by the fryer in \( k \) seconds. Thus, after \( T \) seconds have elapsed, 3 can be turned off.

1. Notation, Terminology, and Conventions

We refer to gates 1, 2, and 3 as simply 1, 2, and 3. We think of 1, 2, and 3 as removing pounds (lbs) from the (conveyor) belt.

Let \( x \) be a nonnegative number. Then \( \text{Fl}(x) \) is the floor function applied to \( x \): \( \text{Fl}(x) \) is the largest whole number that is less than or equal to \( x \). For example, \( \text{Fl}(5) = 5 \), \( \text{Fl}(11.9999) = 11 \), and \( \text{Fl}(3.371) = 3 \).

An expression like \( 7\text{Fl}(x) \) means do 7 times \( \text{Fl}(x) \).

2. How long 3 needs to be on to get it all packaged

2.1. Assumptions made by the model. In the model presented in this note, we make the following assumptions.

(1) We assume that the following times are all the same: the time at which the fryer puts out the first lbs; the time at which 1 opens for the first time; the time at which 2 opens for the first time; and the time at which 3 opens for the first time.

(2) We assume that 1 and 2 always remove the maximum amount that each can remove; that is, we’re assuming that the belt is always delivering at least the amount that 1 and 2 can remove. This assumption should be valid for most of the time that the fryer is running, assuming that its output is fast enough.

(3) We assume that during the 2 secs that 1 or 2 is open, the lbs are not advancing along the belt; we assume that while 1 and 2 are open, the position of the frontmost lbs on the belt is fixed.

(4) We assume that it is accurate to assume that the parameter \( s \), defined in the next subsection, is big enough, so that, after 1 opens and then closes and then the product goes on and arrives at 2, then 2 immediately opens and removes this product. Thus, we are assuming that \( s \) is equal to \((2+8)-2 = 8\) or \( 8+10 = 18 \) or 28 or 38, etc. Similarly, we assume that \( r \) (defined later) is such that, after the fryer starts producing lbs, once the lbs arrive for the first time at 1, then 1 immediately opens and begins removal. Thus, we are assuming that \( r \) is a multiple of 10, so that \( r \) is 10, 20, or 30, etc. Then,

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we can assume that after \( r + 2 + s + 2 + t \) secs (see the next subsection for the meaning of the parameters), the lbs arrive at 3 for the first time. Then 3 begins removal. We assume that, as long as 3 is on, there are lbs at 3 so that 3 can remove the maximum amount of lbs that it is able to. Maybe these are the least accurate assumptions made by this model.

(5) We assume that as long as 1 and 2 are on, they repeat the cycle of 2 secs open, then 8 secs closed. Also, we assume that as long as 3 is on, it is open.

2.2. The parameters that the model takes as input. We define our parameters as follows:

(1) \( p \) is the number of lbs per second that the fryer produces;
(2) we want to know what the model says after the system has been running for \( k \) seconds;
(3) it takes \( r \) seconds for the lbs to go from being deposited by the fryer onto the belt to arriving at 1;
(4) it takes \( s \) seconds for the lbs to go along the belt from 1 to 2;
(5) it takes \( t \) seconds for the lbs to go along the belt from 2 to 3;
(6) 1 removes \( x \) lbs/sec from the belt;
(7) 2 removes \( y \) lbs/sec from the belt; and
(8) 3 removes \( z \) lbs/sec from the belt.

2.3. Presentation of the model. The system runs for \( k \) seconds. After just \( r \) seconds, the lbs arrive at 1, so that for a total of \( (k - r) \) secs, lbs are arriving at 1. Then \( \text{Fl}\left(\frac{k-r}{10}\right) \) is the number of times that 1 repeats the 10 sec cycle of opening and closing. Let

\[
R_1 = (k - r) - 10\text{Fl}\left(\frac{k-r}{10}\right),
\]

which is the number of secs that 1 is on, excluding the time it has to repeatedly perform the 10 sec cycle and the \( r \) secs it takes for the lbs to arrive at 1 for the first time. Then, letting \( N_1 \) be the number of lbs that 1 removes from the belt during the \( k \) seconds, we have

\[
N_1 = 2x\text{Fl}\left(\frac{k-r}{10}\right) + \begin{cases} xR_1 & \text{if } R_1 \leq 2 \\ 2x & \text{if } R_1 > 2. \end{cases}
\]

Similarly, after \( (r+2+s) \) secs, the lbs arrive at 2 for the first time; for a total of \( k-(r+s+2) \) secs, 2 has lbs arriving at its gate. Each time 1 opens, there is a 2 sec delay in the lbs being conveyed to 2. But the vibratory distribution conveyor closes up the gap that is caused by the delay, so we ignore this effect. Then \( \text{Fl}\left(\frac{k-r-s-2}{10}\right) \) is the number of times that 2 repeats the 10 sec cycle of opening and closing. We let \( R_2 \) play the role of \( R_1 \) above, so that

\[
R_2 = (k - r - s - 2) - 10\text{Fl}\left(\frac{k-r-s-2}{10}\right).
\]

Thus, if \( N_2 \) is the number of lbs that 2 removes from the belt during the \( k \) seconds, we have

\[
N_2 = 2y\text{Fl}\left(\frac{k-r-s-2}{10}\right) + \begin{cases} yR_2 & \text{if } R_2 \leq 2 \\ 2y & \text{if } R_2 > 2. \end{cases}
\]
The fryer has produced $pk$ lbs during the $k$ secs. 1 and 2 have removed $N_1 + N_2$ lbs during the $k$ secs. This leaves $(pk - N_1 - N_2)$ lbs that must be removed by 3. 3 needs $\frac{pk - N_1 - N_2}{z}$ secs to remove all these remaining lbs. It takes 

$$(r + 2 + s + 2 + t) = (r + s + t + 4)$$

secs for the lbs to first arrive at 3; that is, the cycle $C$ of (a) fryer sending lbs to 1, (b) 1 removing for 2 secs, (c) then sending to 2, (d) 2 removing for 2 secs, and (e) then going on to 3 takes $(r + s + t + 4)$ secs. After $(r + s + t + 4)$ secs, 3 begins removal for the first time. Then, for $(r + s + t + 4)$ secs, it does removal while $C$ happens again. Then, for another $(r + s + t + 4)$ secs, it does removal while $C$ happens a third time. And so on. Therefore, since 3 is on and open from the beginning, 3 needs to be on for a total of only

$$T = (r + s + t + 4) + \frac{pk - N_1 - N_2}{z}.$$ seconds. After 3 has been on for $T$ seconds, it is no longer needed and so it can be turned off.

2.4. Conclusion. Let 

$$\epsilon_1 = \begin{cases} xR_1 & \text{if } R_1 \leq 2 \\ 2x & \text{if } R_1 > 2, \end{cases}$$

and let 

$$\epsilon_2 = \begin{cases} yR_2 & \text{if } R_2 \leq 2 \\ 2y & \text{if } R_2 > 2. \end{cases}$$

Then 3 can be turned off after

$$T = (r + s + t + 4) + \frac{1}{z} \left( pk - 2x\text{Fl}\left( \frac{k - r}{10} \right) - 2y\text{Fl}\left( \frac{k - r - s - 2}{10} \right) - \epsilon_1 - \epsilon_2 \right)$$

seconds.

2.5. A computation of $T$ using the data from the pdf file that you sent me. Using the data that you gave me:

(1) $p = \frac{30}{60} = \frac{1}{2}$ lbs/sec;
(2) let the system run for one hour, so that $k = 60 \cdot 60 = 3600$ secs;
(3) suppose that $r = 30$ secs;
(4) suppose that $s = 8$ secs;
(5) suppose that $t = 9$ secs;
(6) since 1 removes 500 lbs in an hour and 1 is only open for $\frac{3600 \cdot 2}{10} = 720$ secs during this hour, we find that $x = \frac{500}{720} = \frac{25}{36}$ lbs/sec;
(7) similarly, $y = \frac{700}{720} = \frac{35}{36}$ lbs/sec; and
(8) $z = \frac{900}{3600} = \frac{1}{4}$ lbs/sec, since 3 is always open.

Then

$$R_1 = (3600 - 30) - 10\text{Fl}\left( \frac{3570}{10} \right) = 0,$$

so that 

$$\epsilon_1 = 0.$$ 

Also,

$$R_2 = (3600 - 30 - 8 - 2) - 10\text{Fl}\left( \frac{3560}{10} \right) = 0,$$
so that \( \epsilon_2 = 0 \).

Thus,
\[
T = (30 + 8 + 9 + 4) + 4 \cdot (1800 - 2 \cdot \frac{25}{36} \cdot 357 - 2 \cdot \frac{35}{36} \cdot 356 - 0 - 0)
\]
\[
= 2498.778 \text{ secs.}
\]

We conclude that, in this example, machine 3 needs to run for 2,498.778 seconds = 41.6463 minutes, and then it can be turned off.